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On Subgroups of Non-Commutative Orthogonal Rhotrix Group

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ABSTRACT

This study investigates the algebraic structure of the non-commutative orthogonal rhotrix group under rhotrix row-column multiplication. The special orthogonal, diagonal orthogonal, and special diagonal orthogonal rhotrix groups are identified as subgroups, and their internal relationships are explicitly characterized through subgroup inclusions and intersections. In particular, it is shown that the orthogonal rhotrix group embeds as a subgroup of the general linear rhotrix group. To the best of our knowledge, this work provides the first systematic subgroup structural analysis of non-commutative orthogonal rhotrix groups. These results clarify the internal organization of orthogonal rhotrix groups and provide a foundational framework for further studies on normal subgroups, quotient structures, and related non-commutative rhotrix construction.

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1.0 Introduction

Rhotrices provide a rhomboidal generalization of matrices and offer an alternative framework for representing algebraic structures beyond the classical rectangular setting. Since their introduction by Ajibade[1] as an extension of the matrix concepts of tersions and noitrets introduced by Atanassov and Shannon[2], rhotrices have attracted increasing attention as underlying objects for algebraic investigation. In particular, several authors [3], [4], [5], [6] have demonstrated that rhotrix sets, when equipped with suitable binary operations, give rise to rich algebraic systems analogous to classical matrix groups.

Recent work by Mohammed and Okon[8] considered the collection of all invertible rhotrices of size n over a field F, together with the row-column method of multiplication, and established a general linear rhotrix group. This development naturally raises the question of whether important subclasses of invertible rhotrices, analogous to those in classical matrix theory, can be isolated and studied independently. One of such subclasses is formed by orthogonal rhotrix group.

Orthogonality introduces additional algebraic structure beyond invertibility. An orthogonal rhotrix satisfies a defining relation of the form $M_n^T \circ M_n = I_n$, which ensures that the inverse of an orthogonal rhotrix coincides with its transpose. This property, familiar from classical orthogonal matrices, leads to strong internal symmetry and motivates the study of orthogonal rhotrices as distinct algebraic object rather than merely subset of the linear rhotrix group.

The present work is inspired by the classical relationship between matrix groups. In matrix theory, the orthogonal group $O_{(n)}$ arises as a subgroup of the general linear group $GL_n(F)$, while the special orthogonal group $SO_{(n)}$ appears as a normal subgroup characterized as the kernel of the determinant homomorphism. These groups play a central role in geometry, physics, and representation theory. Our aim is to extend this well-established paradigm to the rhotrix setting and to examine how much of the classical subgroup structure persists in the non-commutative rhotrix framework.

In this paper, we adopt the row-column method of rhotrix multiplication to study the algebraic structure of the non-commutative orthogonal rhotrix group, denoted by $OR_n(F)$. Rather than focusing on general definitions, which are presented in the preliminaries section, we concentrate on the structural properties of this group and its principal subgroups. In particular, we identify and analyze the special orthogonal rhotrix group, the diagonal orthogonal rhotrix group, and the special diagonal orthogonal rhotrix group, and we clarify their inclusion and intersection relationships.

The objectives of this paper are threefold: first, to establish rigorously that the set of orthogonal rhotrices forms a group under the row-column multiplication; second, to characterize its key subgroups and their internal relationships, including normality and intersection structure; and third, to show that the orthogonal group embeds naturally as a subgroup of the general linear rhotrix group. These results extend classical group-theoretical ideas to the rhotrix setting and provide a foundational framework for further investigations into quotient structure, Lie-type rhotrix groups, and potential applications in geometry and mathematical physics.

2.0 Prelimnaries

The following definitions provide the foundational framework for the discussion in the subsequent sections.

2.1 Rhotrices

2.1.1 Definition of a Rhotrix

A rhotrix is a rhomboidal arrangement of the form

$$\Psi = \left\langle \begin{array}{ccc} & \alpha & \\ \beta & h(\Psi) & \gamma \\ & \delta & \end{array} \right\rangle$$

Where $\alpha, \beta, h(\Psi), \gamma, \delta \in R$ is a rhotrix. The entry at the centre of a rhotrix denoted by $h(\Psi)$ is called the heart.

2.2 Operations on Rhotrices

2.2.1 Addition of Rhotrices

The sum of two rhotrices Ψ and Φ was defined as

$$\Psi + \Phi = \left\langle \begin{array}{ccc} & \alpha & \\ \beta & h(\Psi) & \gamma \\ & \delta & \end{array} \right\rangle + \left\langle \begin{array}{ccc} & \iota & \\ \kappa & h(\Phi) & \lambda \\ & \mu & \end{array} \right\rangle$$

$$= \left\langle \begin{array}{ccc} & \alpha + \iota & \\ \beta + \kappa & h(\Psi) + h(\Phi) & \gamma + \lambda \\ & \delta + \mu & \end{array} \right\rangle$$

It was noted that addition is commutative.

2.2.2 Row-column Multiplication of Rhotrices

Using the row-column multiplication method of multiplication, a rhotrix Ψ of size n is decomposed into an ordered pair of matrices $[a_{ij}]$ and $[c_{lk}]$, corresponding to its major and minor entries respectively. Hence $\Psi_n = \langle a_{ij}, c_{lk} \rangle = A_{tt} C_{(t-1)(t-1)}$.

$$\Psi_n \circ \Phi_n = \langle a_{i1j1}, c_{l1k1} \rangle \circ \langle b_{i2j2}, d_{l2k2} \rangle$$

$$= \langle \textstyle\sum_{i2j1}^{t} \left(a_{i1j1}, b_{i2j2} \right), \sum_{l2k1}^{t-1} \left(c_{l1k1}, d_{l2k2} \right) \rangle$$

It should be noted that the row-column multiplication method is non-commutative but associative. The identity element of a rhotrix of size n was also given as

$$I_n = \langle I_{tt}, I_{(t-1)(t-1)} \rangle$$

$$= \left\langle \begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & 1 & 0 & & \\ & - & - & - & - & - & \\ 0 & - & - & 1 & - & - & 0 \\ & - & - & - & - & - & \\ & & 0 & 1 & 0 & & \\ & & & 1 & & & \end{array} \right\rangle$$

An alternative method for rhotrix multiplication known as "heart-based multiplication" was proposed in [1].

2.2.3 Determinant of Rhotrices

The determinant of an n-sized rhotrix is defined as:

$\det(\Psi_n) = \det\langle a_{ij}, c_{lk}\rangle = \det(A_{tt}).\det(C_{(t-1)(t-1)})$ and

$\det(\Psi_n \circ \Phi_n) = \det(\Psi_n) \circ \det(\Phi_n) = \det(\Psi_n).\det(\Phi_n)$

2.2.4 Inverse of Rhotrices

Invertibility of the rhotrix $\Psi_n = \langle a_{ij}, c_{lk}\rangle$ is guaranteed whenever its constituent matrices a_{ij} and c_{lk} are invertible. Consequently, if $\Psi_n^{-1} = \langle q_{ij}, r_{lk}\rangle$, then q_{ij} and r_{lk} are the inverse entries of matrices A_{tt} and $C_{(t-1)(t-1)}$ respectively. Invertibility of Ψ_n is equivalent to the condition $\det(\Psi_n) \neq 0$.

2.2.5 Transpose of Rhotrices

It was defined in [11] that for any rhotrix $\Psi_n = \langle a_{ij}, c_{lk}\rangle$, its transpose $\Psi_n^T = \langle q_{ji}, r_{kl}\rangle$. Thus $(\Psi_n \circ \Phi_n)^T = (\Phi_n)^T \circ (\Psi_n)^T$.

2.3 The Orthogonal Rhotrix Set.

Let F be a field. Let $R_n(F)$ denotes the set of all rhotrices of size n with entries from F defined as

$$R_n(F) = \left\{ \left\langle \begin{array}{ccccccc} & & & a_{11} & & & \\ & & a_{21} & c_{11} & a_{12} & & \\ & - & - & - & - & - & \\ a_{t1} & - & - & - & - & - & a_{1t} \\ & - & - & - & - & - & \\ & & a_{t(t-1)} & c_{(t-1)(t-1)} & a_{(t-1)t} & & \\ & & & a_{tt} & & & \end{array} \right\rangle : a_{ij}, c_{lk} \in F \right\},$$

where $1 \leq i, j \leq t$, $1 \leq l, k \leq t-1$; $t = \frac{1}{2}(n+1)$ and $n \in 2Z^+ + 1$.

The orthogonal rhotrix set denoted as $OR_n(F)$ is the collection of all rhotrices whose transpose multiplied by themselves gives the identity rhotrix. The set:

$$OR_n(F) = \{M_n \in R_n(F) : M_n^T M_n = I_n\}$$

where:

- $OR_n(F)$ is the collection of orthogonal rhotrices with entries from F.
- M_n is the collection of all rhotrices of a fixed size n.
- M_n^T denotes the transpose of M_n, and
- I_n is the identity rhotrix of the same size.

3.0 The Non-commutative Orthogonal Rhotrix Group

In this section, we establish that the set of orthogonal rhotrices forms a group under the row-column multiplication and examine its algebraic properties. Throughout this section, multiplication of rhotrices is carried out using the row-column rule, which is the standard multiplication adopted in rhotrix theory. In [10], it is stated that rhotrix multiplication under row-column method is not commutative in general, that is

$$M_n N_n \neq N_n M_n \quad \forall\, M_n, N_n \in OR_n(F)$$

Theorem 3.1

Let $OR_n(F)$ denote the set of orthogonal rhotrices and let ° denote the multiplication induced by the row-column method. Then, the algebraic structure $(OR_n(F), \circ)$ forms a non-commutative rhotrix group of size n over F.

Proof

we shall show that the following group axioms are satisfied.

- Closure:

Let M_n, $N_n \in OR_n(F)$. By definition of orthogonality,

$$M_n^T \circ M_n = I_n \quad \text{and} \quad N_n^T \circ N_n = I_n$$

Consider the product $M_n \circ N_n$. Using the properties of transpose under row-column multiplication, we have

$$(M_n \circ N_n)^T \circ (M_n \circ N_n) = N_n^T \circ M_n^T \circ M_n \circ N_n = N_n^T \circ I_n \circ N_n = N_n^T \circ N_n = I_n$$

Hence, $M_n \circ N_n$ is orthogonal and therefore $M_n \circ N_n \in OR_n(F)$. Thus, $OR_n(F)$ is closed under "°".

- Associativity:

The row-column method of rhotrix multiplication is associative as shown in [10]. Hence, for all $M_n, N_n, P_n \in OR_n(F)$:

$$M_n \circ (N_n \circ P_n) = (M_n \circ N_n) \circ P_n$$

- Identity:

Let I_n denotes the identity rhotrix. Since $I_n^T = I_n$ and $I_n^T \circ I_n = I_n$.

It follows that $I_n \in OR_n(F)$ and I_n acts as identity element under "°"

- Inverse:

Let $M_n \in OR_n(F)$. Since M_n is orthogonal, we have

$$M_n^T \circ M_n = I_n \quad \text{and} \quad M_n \circ M_n^T = I_n$$

Thus, M_n^T is both a left and right inverse of M_n with respect to "°". Hence every element of $OR_n(F)$ has an inverse in $OR_n(F)$.

Having verified closure, associativity, identity and inverses, we conclude that $(OR_n(F), \circ)$ is a group.

3.1 Illustrative Example

The following example presents a finite set of explicitly given orthogonal rhotrices and uses a Cayley table to illustrate the group operation under row-column multiplication.

Example 3.1

Let $FOR_5(Z_2)$ denotes the finite set of orthogonal rhotrices of size 5 with entries from Z_2. Let us denote the elements of $FOR_5(Z_2)$ as follows:

$$\Psi_1 = \left\langle \begin{array}{ccccc} & & 1 & & \\ & 0 & 1 & 0 & \\ 0 & 0 & 1 & 0 & 0 \\ & 0 & 1 & 0 & \\ & & 1 & & \end{array} \right\rangle, \Psi_2 = \left\langle \begin{array}{ccccc} & & 1 & & \\ & 0 & 1 & 0 & \\ 0 & 0 & 0 & 0 & 0 \\ & 1 & 1 & 1 & \\ & & 0 & & \end{array} \right\rangle, \Psi_3 = \left\langle \begin{array}{ccccc} & & 0 & & \\ & 1 & 1 & 1 & \\ 0 & 0 & 0 & 0 & 0 \\ & 0 & 1 & 0 & \\ & & 1 & & \end{array} \right\rangle$$

$$\Psi_4 = \left\langle \begin{array}{ccccc} & & 0 & & \\ & 0 & 1 & 1 & \\ 1 & 0 & 0 & 0 & 0 \\ & 0 & 1 & 1 & \\ & & 0 & & \end{array} \right\rangle, \Psi_5 = \left\langle \begin{array}{ccccc} & & 0 & & \\ & 1 & 1 & 0 & \\ 0 & 0 & 0 & 0 & 1 \\ & 1 & 1 & 0 & \\ & & 0 & & \end{array} \right\rangle, \Psi_6 = \left\langle \begin{array}{ccccc} & & 0 & & \\ & 0 & 1 & 0 & \\ 1 & 0 & 1 & 0 & 1 \\ & 0 & 1 & 0 & \\ & & 0 & & \end{array} \right\rangle$$

The following Cayley table is provided solely for illustration of Theorem 3.1 and do not form part of its proof.

Table 1: Cayley table illustrating the row-column multiplication of a finite set of orthogonal rhotrices.

$\circ$	Ψ_1	Ψ_2	Ψ_3	Ψ_4	Ψ_5	Ψ_6
Ψ_1	Ψ_1	Ψ_2	Ψ_3	Ψ_4	Ψ_5	Ψ_6
Ψ_2	Ψ_2	Ψ_1	Ψ_4	Ψ_3	Ψ_6	Ψ_5
Ψ_3	Ψ_3	Ψ_5	Ψ_1	Ψ_6	Ψ_2	Ψ_4
Ψ_4	Ψ_4	Ψ_6	Ψ_2	Ψ_5	Ψ_1	Ψ_3
Ψ_5	Ψ_5	Ψ_3	Ψ_6	Ψ_1	Ψ_4	Ψ_2
Ψ_6	Ψ_6	Ψ_4	Ψ_5	Ψ_2	Ψ_3	Ψ_1

The Cayley table confirms that the selected orthogonal rhotrices are closed under multiplication and that inverses occur within this finite subset in agreement with Theorem 3.1.

It is known (see Mohammed and okon[7]) that the set of all invertible rhotrices of size n over a field F, equipped with row-column multiplication, forms a group.

4.0 Embedding and Subgroup Structure of the Orthogonal Rhotrix Group

This section establishes the embedding of the orthogonal rhotrix group into the general linear rhotrix group and examines the internal subgroup structure of the orthogonal rhotrix group. Particular attention is given to special orthogonal, diagonal orthogonal, and special diagonal orthogonal rhotrix subgroups, together with their inclusion and intersection relationships.

Lemma 4.1 (Invertibility of Orthogonal Rhotrices)

Every orthogonal rhotrix is invertible, and its inverse is given by its transpose.

Proof

Let $M_n \in OR_n(F)$. By definition of orthogonality,

$$M_n^T \circ M_n = I_n \quad \text{and} \quad M_n \circ M_n^T = I_n$$

Hence, M_n possesses a two-sided inverse in $\in OR_n(F)$, namely M_n^T. Therefore, every orthogonal rhotrix is invertible.

Theorem 4.1 (Embedding Theorem)

The orthogonal rhotrix group $(OR_n(F), \circ)$ embeds as a subgroup of the general linear rhotrix group $(GR_n(F), \circ)$.

Proof

By Lemma 3.1, every orthogonal rhotrix is invertible. Hence

$$OR_n(F) \subseteq GR_n(F)$$

We define $\propto$: $(OR_n(F), \circ) \rightarrow (GR_n(F), \circ)$ by $\propto(M_n) = M_n$ for all $M_n \in OR_n(F)$.

We show that $\propto$ is injective homomorphism.

Homomorphism.

For any $M_n, N_n \in OR_n(F)$, Then

$$\propto(M_n \circ N_n) = M_n \circ N_n = \propto(M_n) \circ \propto(N_n).$$

Injectivity.

Suppose $\propto(M_n) = \propto(N_n)$. Then $M_n = N_n$, and hence $\ker(\propto) = I_n$. Therefore, $\propto$ is injective.

Since $\propto$ is an injective homomorphism, it follows that $OR_n(F)$ embeds as a subgroup of $GR_n(F)$.

4.1 A General Subgroup Criterion

To avoid repetitive subgroup proofs, we establish general result.

Proposition 4.1

Let $H \subseteq OR_n(F)$ be a non-empty subset such that:

- H is closed under row-column multiplication and
- H is closed under transpose.

Then H is a subgroup of $OR_n(F)$.

Proof

Let $M_n \in H$. Since $M_n \in OR_n(F)$, orthogonality implies $M_n^{-1} = M_n^T$.

By assumption, $M_n^T \in H$, hence H contains inverses. Closure under multiplication is given, and associativity is inherited from $OR_n(F)$. Therefore, H is a subgroup of $OR_n(F)$.

Definition 4.1 (Diagonal Orthogonal Rhotrix)

A diagonal orthogonal rhotrix over a field F is a rhotrix of size n that is vertically diagonal (that is, all its off-diagonal entries are zero) and orthogonal, with each diagonal entry equal to either 1or -1. It is denoted by $DOR_n(F)$. Thus:

$$DOR_n = \left\{ \left\langle \begin{array}{ccccccccc} & & & & a_{11} & & & & \\ & & & 0 & c_{11} & 0 & & & \\ & - & - & & - & & - & - & \\ 0 & - & - & & - & & - & - & 0 \\ & - & - & & - & & - & - & \\ & & & 0 & c_{(t-1((t-1)} & 0 & & & \\ & & & & a_{tt} & & & & \end{array} \right\rangle : a_{ij}, c_{lk} \in \{+1,-1\} \right\}$$

Since the set of diagonal orthogonal rhotrices is closed under the multiplication and transpose, it follows Proposition 4.1 that $DOR_n(F)$ is a subgroup of $OR_n(F)$. Morerover, diagonal rhotrices commute under row-column multiplication; therefore, $DOR_n(F)$ is an abelian subgroup of $OR_n(F)$.

Definition 4.2 (Special Orthogonal Rhotrix Group)

Special orthogonal rhotrix $SOR_n(F)$ is defined to be subset of $OR_n(F)$ consisting of those orthogonal rhotrices whose determinant equals 1. Thus

$$SOR_n(F)=\{M_n \in OR_n(F): \det(M_n)=1\}$$

Proposition 4.2 (Determinant Homomorphism)

Let det: $GR_n(F) \to F^X$ denote the determinant map on the general linear rhotrix group. Then the restriction

$$\det: OR_n(F) \to F^X$$

Proof

Let M_n, $N_n \in OR_n(F)$. By the multiplicativity of the rhotrix determinant,

$$\det(M_n \circ N_n) = \det(M_n)\det(N_n)$$

Lemma 4.2 (Determinant of Orthogonal Rhotrices)

If $M_n \in OR_n(F)$, then $\det(M_n)^2=1$. In particular,

$$\det(M_n) \in \{1, -1\}.$$

Proof

$$M_n^T \circ M_n = I_n$$

$$\det(M_n^T)\det(M_n) = \det(I_n) = 1$$

Theorem 4.2 (Characterization of the Special Orthogonal Rhotrix Group)

The special orthogonal rhotrix group is precisely the kernel of the determinant homomorphism on $OR_n(F)$, that is,

$$SOR_n(F) = \ker(\det \mid OR_n(F))$$

Proof

By definition, $\ker(\det \mid OR_n(F)) = \{M_n \in OR_n(F): \det(M_n)=1\}$

Hence,

$$SOR_n(F) = \ker(\det \mid OR_n(F))$$

Corollary 4.1

The special orthogonal rhotrix group is a normal subgroup of the orthogonal rhotrix group :

$$SOR_n(F) \trianglelefteq OR_n(F)$$

Reason: Kernel of homomorphism are normal

Definition 4.3 (Special Diagonal Orthogonal Rhotrix)

A special diagonal orthogonal rhotrix over a field F is a rhotrix size n that is diagonal, orthogonal and has determinant equals 1. It is denoted as $SDOR_n(F)$. Thus

$$SDOR_n(F)=\{A_n \in DOR_n(F): \det(A_n)=1\}$$

Since the set of special diagonals orthogonal rhotrices is closed under multiplication and transpose, proposition 4.1 applies. It is also abelian since it is contained in the abelian group $DOR_n(F)$.

4.2 (Subgroup Relationship)

The subgroup relationship may be summarized as:

Special diagonal orthogonal rhotrix group ⊆ Special orthogonal rhotrix group⊆ Orthogonal rhotrix group.

Special diagonal orthogonal rhotrix group ⊆ Diagonal orthogonal rhotrix group ⊆ Orthogonal rhotrix group.

Proposition 4.3 (Intersection Characterization)

The special diagonal orthogonal rhotrix group is the intersection of the special orthogonal rhotrix group and the diagonal orthogonal rhotrix group. Thus:

$$SDOR_n(F)= SOR_n(F) \cap DOR_n(F)$$

Proof

Let $M_n \in SDOR_n(F)$. Then M_n is diagonal, orthogonal, and satisfies $\det(M_n)=1$. Hence, $M_n \in DOR_n(F)$ and $M_n \in SOR_n(F)$, implying:

$$M_n \in SOR_n(F) \cap DOR_n(F)$$

Conversely,

Let $M_n \in SOR_n(F) \cap DOR_n(F)$. Then, M_n is diagonal and orthogonal, and $\det(M_n)=1$. By definition $M_n \in SDOR_n(F)$.

Therefore,

$$SDOR_n(F)= SOR_n(F) \cap DOR_n(F)$$

Corollary 4.1 (Structural Properties)

- $SOR_n(F)$ is a normal subgroup of $OR_n(F)$
- $DOR_n(F)$ is an abelian subgroup of $OR_n(F)$
- $SDOR_n(F)$ is an abelian subgroup of $OR_n(F)$

Proof

- $SOR_n(F)$ is the kernel of a homomorphism, hence it is normal.
- $DOR_n(F)$ commute under row-column multiplication
- $SDOR_n(F) \subseteq DOR_n(F)$; thus, it is abelian.

4.3 Subgroup Lattice Diagram

These subgroup relationships induce a natural lattice structure among the orthogonal rhotrix subgroups.

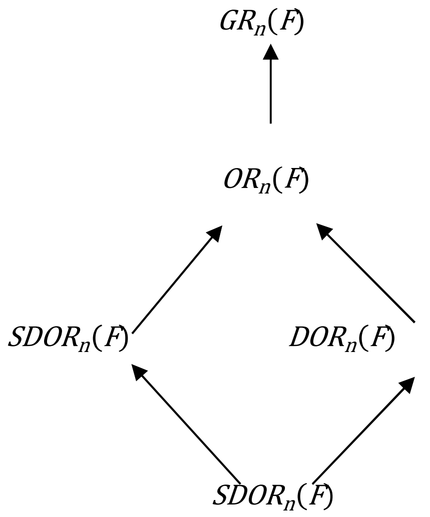

Fig 1:Subgroup lattice showing inclusion relationships among the orthogonal rhotrix group and its distinguished subgroups.

The figure shows the subgroup lattice of the orthogonal rhotrix group $OR_n(F)$, showing the inclusion and intersection relationships among the special orthogonal rhotrix group $SOR_n(F)$, the diagonal orthogonal rhotrix group $DOR_n(F)$, and the special diagonal orthogonal rhotrix group $SDOR_n(F)$.

4.0 Conclusion

This work establishes the non-commutative orthogonal rhotrix group as a well-defined algebraic structure under row-column multiplication and situates it naturally within the general linear rhotrix group via an explicit embedding. The identification of special orthogonal, diagonal orthogonal, and special diagonal orthogonal rhotrix subgroups, together with their inclusion and intersection relationships, reveals internal lattice structure analogous to that of the classical orthogonal group, while reflecting the distinctive non-commutative nature of rhotrices.

Beyond confirming group theoretical properties, the embedding and kernel characterizations clarify how orthogonality constraints interact with invertibility in the rhotrix setting. In particular, viewing the special orthogonal rhotrix group as the kernel of the determinant homomorphism highlights a fundamental structural parallel with classical matrix groups, thereby strengthening the conceptual foundation of orthogonal rhotrix theory.

These results open several natural directions for future research. The established subgroup and kernel structures provide a basis for studying quotient rhotrix groups and their algebraic properties. Further work may also investigate Lie-type rhotrix groups, representation theory and possible application to geometry and physics, where orthogonality-preserving transformation play a central role.

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