A Stochastic Model of Indigenous Language Extinction in Nigeria

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Abstract

The need to effectively capture indigenous language decline is explored in this study. Language decline was conceptualized as a Poisson process in order to study the dynamics. Expressions and estimates were derived for the intergenerational transmission probability, the probability of eventual extinction and the mean number of transfers per family. For model validation, a household survey was conducted in Salem City, Warri, Nigeria. Demographic information about the community was elicited through a multi-dimensional questionnaire, which also provided information on the actual number of children who were able to imbibe their heritage language, across two generations. A massive demographic shift was observed in the family sizes over the two generations studied. There was also a general decline in intergenerational transmission, pointing to the possible extinction of such languages in generations to come. Our study also showed that a great proportion of parents in the sampled community have indigenous language ability but hardly impart such to their offspring. The findings further indicated that some indigenous languages are under serious threat of extinction if conscious steps are not taken to arrest the decline. This model can be used to ascertain the status of any indigenous language globally.

Keywords: Poisson process, branching process, language transmission, extinction probability.

1.0 Introduction

The fact that languages have been going extinct has been established over the years by linguists and historians [1]. Although languages could go extinct due to random loss of effective speakers through intergenerational transmission, the time to extinction in the absence of language revitalization measures, is of interest. The prestige of English language and other ex-colonial languages in Africa has been on the increase, possibly due to their association with modernity, technological and economic advancement, information flow and their global reach [2]. Many people in Africa, particularly the elite, have come to consider the ex-colonial languages as central to the economic and technological development of the continent. Even at a personal level, many parents would like to see their children speak fluent English or French or Portuguese. Many of these parents would not even mind if their children had limited fluency in their own mother tongue, as these languages are not associated with social advancement, job opportunities or the wider world [2].

The minority indigenous languages are at the bottom of the hierarchical structure and are, in most countries, marginalized in that they are not accorded any public function or socio-economic prestige. Understanding the process underlying language extinction is an important problem in studies of language maintenance. As an indigenous language evolves in a large finite population, there is the possibility of insufficient transmission for it to stay alive. If there is no definite teaching of the language, the expected number (λ) of offspring that are interested in their indigenous language is small. Therefore in finite time, the number of effective speakers can become so small that there is insufficient transmission to prevent the language from extinction. Thus in the absence of language revitalization programmes, the language dies out. The problem of language endangerment and extinction has taken a more worrisome dimension in recent generations. This has occasioned the loss of some languages and portends a grim future for many of Nigeria's indigenous languages, as well as their embedded cultures. Currently in Nigeria, many dialects of larger language groups are even almost dead, as several factors contribute to the inability of parents to transmit these dialects to their children, coupled with the unwillingness/ lack of interest of the children to imbibe those dialects.

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Few of the papers that explored modelling language extinction are mentioned. The dynamics of language death was modelled using an ordinary differential equation (ODE) model, with one of the parameters in the model as the relative status of the language [3]. The model was shown to be appropriate for scenarios where there is language competition between two languages. Others applied operations research methodology in modelling the interaction between unilingual and bilingual populations, and applied the model to instances of modern Canada and Wales [4]. A reaction-diffusion model was proposed to analyze the dynamics of interactions of a population with two monolingual groups and a group that is bilingual in the two languages [5]. The results established that demographic factors, such as population growth or population dispersal, play an important role in the competition dynamics.

The interaction between language evolution and demography was explored using three different approaches: analytical modelling, agent-based analytical modelling and agent-based cognitive modelling [6]. The author concluded that the agent-based cognitive models allowed for the most detailed and realistic simulations. A mathematical model was developed for language extinction, accommodating the mechanisms that influence language competition, and how these mechanisms are influenced in the various revitalization policies [7]. The model revealed that it is possible to preserve a minority language but that continued long-term interventions will likely be necessary. However, the model is a social evolution model based on ordinary differential equations, and is highly deterministic.

This study is motivated by the need to effectively capture the scenario of indigenous language decline, which is a result of several competing factors that affect the transmission from one generation to the next. Poisson processes theory provides a simple but powerful tool for modelling, as applied to language depletion. Poisson processes constitute the simplest and most widely applied among the class of renewal processes [8]. In its simplicity lies the versatility of Poisson processes. A Poisson process is a renewal process in which the interarrival times are exponentially distributed random variables. Poisson processes constitute the simplest and most widely applied among the class of renewal processes [8]. In its simplicity lies the versatility of Poisson processes. A Poisson process is a renewal process in which the interarrival times are exponentially distributed random variables. The parameter λ is called the rate of the process, and for any interval of length t, λt is the number of arrivals in that interval. There are several important results and theorems establishing the uniqueness of Poisson processes. There is a link between the interarrival times and the counting process. The fact is that if the interarrival times are exponentially distributed, then the process counting the number of arrivals up to and including time t is distributed Poisson. The exponential and Poisson distribution are thus a continuous-discrete couple, as far as renewal processes are concerned.

The ability and desire of parents to transmit their indigenous languages and cultures to their children has been largely affected by various socio-economic, religious and political factors. This makes it impossible for such children to be able to teach their mother tongue to their offspring when they grow up and have their own families. This poses a grave danger to the survival of such languages with low transmission rate between generations. The present study is therefore intended to investigate the problem of language endangerment and extinction. The idea is to propose a stochastic model that effectively captures the dynamics of language change with a view to exposing the key parameters that impact the process of language extinction, as well as language revitalization and maintenance policies. The objectives of the study are to develop a stochastic model of language extinction using established principles from the theory of Poisson processes; and undertake a survey in a Nigerian community to ascertain the level of indigenous language decline, using the developed model.

2.0 Materials and Methods

2.1 Questionnaire Design

A household survey was conducted in Salem City, a community in Warri, Nigeria in which a multi-dimensional questionnaire was administered on most of the households in the community. This was in order to study the dynamics of indigenous language decline in the community from one generation to the other. The questionnaire was structured in such a way that each respondent could provide information on the language speaking ability of his/ her siblings and his/ her children. The information provided by respondents would yield insights about the language pattern of the family over two generations. Language ability among male and female children was also targeted, as well as ability to speak the language of the mother in inter-ethnic marriages.

The level of intergenerational transmission of the language from parents to children could be assessed over the two generations. A careful study of the generational language transmission rates will provide a sneak view of the dynamics of indigenous language decline. The proficiency of indigenous language was also gauged from language understanding (lowest level) to ability to read and write in the language. The frequency of use of the indigenous language in both private and public domain was also captured by the questionnaire. Data processing of the questionnaire was accomplished by the US Census Bureau's CSPro software program.

2.2 Poisson Process of Language Decline

It is assumed that the counting process of the births into a community is Poisson with parameter μ. However, due to various reasons, not all children born in the community will learn to speak the indigenous language, as the language of primary communication is another language (English or Pidgin English, in the Nigerian situation). Thus, the children born into the community form the input for the process counting the number of children who assimilate the indigenous language. With probability p, a child learns his mother tongue, and with probability 1-p, the child does not learn the language.

Let the counting process of births into the community be denoted by {N(t), t >0} with rate μ. Let $\{N_1(t), t > 0\}$ denote the process of births who acquire indigenous language ability and $\{N_2(t), t > 0\}$, the process specifying births who could not acquire indigenous language ability. It is also assumed that the community consists of members from k families. Each birth is switched with probability p to $\{N_1(t), t > 0\}$ and with probability (1-p) to $\{N_2(t), t > 0\}$. Each arrival is switched independently of each other and independently of the arrival epochs.

It may be helpful to visualize the scenario as the combination of two independent processes. The first is the Poisson process of rate μ and the second is a Bernoulli process$\{X_n;\ n \geq 1\}$, where $\Pr(X_n = 1) = p$ and $\Pr(X_n = 2) = 1 - p$. The nth arrival of the Poisson process is, with probability p, tagged as type 1 arrival (X_n=1) and with probability 1-p, it is tagged as type 2 (X_n=2). Let $Z_i(t)$ denote the number of children born into the ith family in the community. Then if there are a total of k families in the community, at time t,

$$N(t) = Z_1(t) + Z_2(t) + \cdots + Z_k(t) \qquad (1)$$

where the $Z_i(t)$'s are independent and identically distributed random variables.
It can be shown that the resulting processes $N_1(t)$ and $N_2(t)$are each Poisson with rates $\mu_1 = p\mu$ and $\mu_2 = (1-p)\mu$, respectively. The process is evaluated at certain points in time t, hence the time epochs are viewed at generational points. It may be viewed as the length of one generation for the different language speaking communities. It is assumed that the language is learnt through interactions at home and in the informal environment, that is, indigenous language acquisition is through intergenerational transmission. The extinction patterns, time of extinction as well as the probability of extinction of the language in the community are investigated, given a particular rate of transmission. The ability to view independent Poisson processes either independently or as splitting of a combined process is a powerful technique for finding solutions to several otherwise less tractable problems.

The Poisson distribution has a characteristic property in relation to the binomial distribution. If the random variable X is distributed Poisson with parameter μ and if a second random variable Y has a conditional distribution, given X, of the form Binomial (X,p), then X and X-Y are independent Poisson variables with respective means $p\mu$ and $(1-p)\mu$ [9]. The colouring theorem as expounded by Kingman [9], which is of huge relevance to the present study, is stated in Theorem 1.

***Theorem 1*:** *Let Π be a Poisson process on S with mean measure μ. Let the points of Π be coloured randomly with k colours, the probability that a point receives the ith colour being p_i and the colours of different points being independent (of one another and of the positions of the points).*
Let Π_i be the set of points with the ith colour. Then the Π_i are independent Poisson processes with mean measures

$$\mu_i = p_i\mu, \quad \sum p_i = 1 \qquad (2)$$

Since the output process of child arrivals forms the input process of the number of language speakers, this is a platooned arrival process [10]. The Poisson process has a number of special properties which make its use and the calculation of associated probabilities often surprisingly simple. A Poisson model is usually the simplest, and in a sense, the most random way in which to describe any particular phenomenon [9]. The most important feature of the Poisson distribution is its additivity [9]. A Poisson process is a counting process for which the interarrival times are independent and identically distributed exponential random variables. The counting process $\{N(t), t \geq 0\}$ is said to be a Poisson proces with rate $\lambda, \lambda > 0$ if:

1. $N(0) = 0$
2. $\{N(t), t \geq 0\}$ has independent increments, and
3. the number of events which occur in any interval of length t is Poisson distributed with parameter λt; that is

$$\Pr\{N(t+s) - N(s) = n\} = \frac{(\lambda t)^n e^{-\lambda}}{n!}, \quad for\ all\ s \qquad (3)$$

Consider a sequence of random variables $X_0, X_1, \cdots, X_n$ where X_n represents the number of children speaking the language in the n[th] generation. It is assumed that the population is initiated by one individual, that is, $X_0 = 1$, and when he dies he is replaced by k individuals with probability p_k, k=0,1,2,.... These individuals behave independently and identically to the parent individual, as do those in subsequent generations.

The number of indigenous language speakers in the (n+1)[th] generation, X_{n+1} is given by:

$$X_{n+1} = \begin{cases} Z_1^n + Z_2^n + \cdots + Z_{X_n}^n & if\ X_n \geq 1 \\ 0 & if\ X_n = 0 \end{cases} \qquad (4)$$

The sequence $\{Z_j^n; j \geq 1\}$ are independent and identically distributed random variables, independent of X_n, and Z_j^n represents the number of offspring of the j[th] individual in the n[th] generation. Let G(s) be the probability generating function for X_n; then:

$$G(s) = \sum_{k=0}^{\infty} p_k s^k = E[s^{X_1}] = e^{-\mu(1-s)} \qquad (5)$$

and

$$G_n(s) = E[s^{X_n}] \qquad (6)$$

$G_1 = G$, and it can be shown that, for all $n \geq 1$,

$$G_{n+1}(s) = G_n(G(s)) \qquad (7)$$

and

$$G_j'(1) = \mu^j \qquad (8)$$

Hence at the j[th] generation, the expected number of persons speaking the language is μ^j.

If the population size m>>μ^j, then the language will become extinct if and only if $\mu < 1$. Thus if m is finite, then the population of the indigenous language speakers will be exterminated in the absence of language revitalization and sustenance programmes. The corresponding moment generating function of the number of speakers of the language is:

$$M_{X_n}(t) = e^{\mu(e^t - 1)} \qquad (9)$$

It can be shown that $Z_i \sim Poisson(\frac{\mu}{k})$, and this may be accomplished using the moment generating function. The distribution of the number of births in the family is therefore distributed Poisson with parameter $\frac{\mu}{k}$. If the assumption of Poisson arrivals is adopted, then the trend of language transfer (intergenerational transmission) over several generations can be determined. From one language to another, there is the need to estimate μ. One way of doing this is to sample from the larger population of language speakers for the specific language under study. For each sampled family, the number of children $(n_i)$and the number of children in the family with the basic ability to speak the language (x_i), i=1,2,...,g, is obtained.

In order to estimate the mean number of transfers per family, the following estimators shall be used:

$$\hat{\mu} = \frac{1}{g} \sum_{i=1}^{g} x_i \qquad (10)$$

and

$$\tilde{\mu} = \sum_{i=1}^{g} f_i x_i \qquad (11)$$

where $f_i = \frac{n_i}{\sum n_i}$

It can be shown that $\hat{\mu}$ and $\tilde{\mu}$ are both unbiased estimators of μ. However, in order to determine which of the estimators is an efficient estimator of μ, the Cramer-Rao lower bound is computed. The variances of the two estimators are, respectively

$$Var(\hat{\mu}) = \frac{\mu}{g} \tag{12}$$

and

$$Var(\tilde{\mu}) = \mu \sum f_i^2 \tag{13}$$

The Cramer-Rao lower bound for the variance of an unbiased estimator $\delta(x_1, x_2, \cdots, x_g)$ of the Poisson parameter μ is given by:

$$Var\left(\delta(x_1, x_2, \cdots, x_g)\right) \geq \frac{1}{-gE\left(\frac{\partial^2}{\partial\mu^2}\ln p(x)\right)} = \frac{\mu}{g} \tag{14}$$

where $p(x)$ is the probability density function of the Poisson distribution.

Thus, comparing the variances of the two estimators with the Cramer-Rao lower bound, it can be seen that $\hat{\mu}$, which is also the maximum likelihood estimator of μ, has the least variance. It is also of interest to estimate the time to extinction of the language. If the principle of stochastic epidemic models is adopted and the system is modelled as a branching process [11], then the following argument follows:

Let $\{X(t), t \geq 0\}$, be the number of individuals speaking the language at time t, $t \geq 0$, and denote by D the number of offspring of a given individual. It is desirable to investigate the possible extinction of X(t) as t grows. First, if there are m initial individuals, then there are on the average $mE(D^j)$ individuals in the j^{th} generation, and it is intuitively clear that the process will become extinct if and only if $E(D) \leq 1$.

For the case where $E(D) > 1$, let q be the extinction probability of the branching process and assume first that m=1. Then by letting D_0 be the number of children of the ancestor, it follows that

$$q = \sum_{k=0}^{\infty} \Pr(extinction | D_0 = k) \Pr(D_0 = k) \tag{15}$$

However, ultimate extinction will occur if and only if all of the (independent) branches generated by these children become extinct, hence

$$q = \sum_{k=0}^{\infty} \mathrm{q}^{\mathrm{k}} \Pr(D_0 = k) \tag{16}$$

Finally, when there are m ancestors the extinction probability is given by q^{m}.

The probability that extinction finally occurs is now considered. It should be noted that $X_n = 0$ implies $X_{n+1} = 0$. Define the event $A_n = \{X_n = 0\}$. $A = \bigcup_{n=1}^{\infty} A_n$ is the event that extinction eventually occurs. The eventual extinction probability is q=Pr(A) and $\Pr(A_n)$ is the probability that the extinction occurs in the n^{th} generation. Since $\Pr(A_n) = G_n(0)$, then:

$$q = \Pr(A) = \lim_{n\to\infty} G_n(0) \tag{17}$$

A relevant theorem, as proved by Ling [12] is stated next:

Theorem 2: *The eventual extinction probability q is the smallest positive root of the equation G(s)=s.*

It is thus possible to solve the equation

$$s - G(s) = s - e^{-\mu(1-s)} = 0 \tag{18}$$

and obtain the smallest positive root, which corresponds to the eventual extinction probability, q. This may be computed for various languages, given estimates of μ.

It may also be of interest to estimate the probable generation of extinction. Let T be the exact extinction time; then

$$\{T = n\} = \{X_n = 0, X_{n-1} > 0\} \quad (19)$$

$\Pr(X_n = 0)$ is the probability of extinction by time n and $\Pr(X_n = 0) - \Pr(X_{n-1} = 0)$ is the probability of extinction at exact time epoch n.

Since $\Pr(X_n = 0) = G_n(0)$ for all n, then;

$$\Pr(T = n) = G_n(0) - G_{n-1}(0) \quad (20)$$

3.0 Results and Data Analysis

From the data elicited from the respondents, estimates of the intergenerational transmission probability ($\hat{p}$), the mean number of speakers per family ($\hat{\mu}$) and the eventual extinction probability (q), were computed. The ethnic groups with greater proportion of inhabitants in the community were Urhobo and Igbo. As in the larger city of Warri, the main language of communication in the community is Pidgin English. Hence all the inhabitants of the community can converse in Pidgin English, while others may also have fluency in one or more indigenous languages. Thus, it is a mainly multilingual community.

Table 1 reflects the mean age at marriage of married respondents, according to their ethnic group. Table 2 presents the estimates of intergenerational transmission probability for the previous generation ($\hat{p}_1$) and present generation ($\hat{p}_2$), as well as estimates of the mean number of generational transfer of speakers of the various languages($\hat{\mu}_1, \hat{\mu}_2, \tilde{\mu}_1, \tilde{\mu}_2$). Estimates are also provided for sub-populations of the ethnic groups (e.g. married with children; and single) and the accompanying extinction probabilities q_1 and q_2. Figures 1 and 2 provide pictorial representations of some demographic characteristics of the sampled population - their ethnic groups and marital status.

Table 1: Mean age at marriage (in years) of married respondents of each ethnic group

Tribe	Males	Females	Overall
Urhobo	26	26	31
Igbo	29	27	28
Yoruba	30	33	32
Others	35	28	30
Overall mean age	**33**	**27**	**30**

Table 2: Estimates of the parameters of the model as well as the extinction probabilities

Language	$\hat{p}_1$	$\hat{\mu}_1$	q_1	$\tilde{\mu}_1$	$\hat{p}_2$	$\hat{\mu}_2$	q_2	$\tilde{\mu}_2$
Urhobo (All)	0.6277	3.69	0.028	5.77	-	-	-	-
Urhobo (Married)	0.9367	6.17	0.002	10.0	0.38	1.67	0.324	1.92
Urhobo (Single)	0.3564	1.89	0.234	2.28	-	-	-	-
Igbo (All)	0.8790	5.45	0.004	5.95	-	-	-	-
Igbo (Married)	0.9687	6.20	0.002	6.78	0.2703	1.00	1	1.41
Igbo (Single)	0.7833	4.70	0.001	5.07	-	-	-	-
Yoruba (All)	1.0	6.14	0.002	6.86	0.5455	1.71	0.302	4.0
Others (All)	0.5398	3.06	0.055	3.51	-	-	-	-
Others (Married)	0.75	3.88	0.022	4.82	0.6545	2.12	0.174	2.87
Others (Single)	0.2817	1.67	0.324	1.59	-	-	-	-

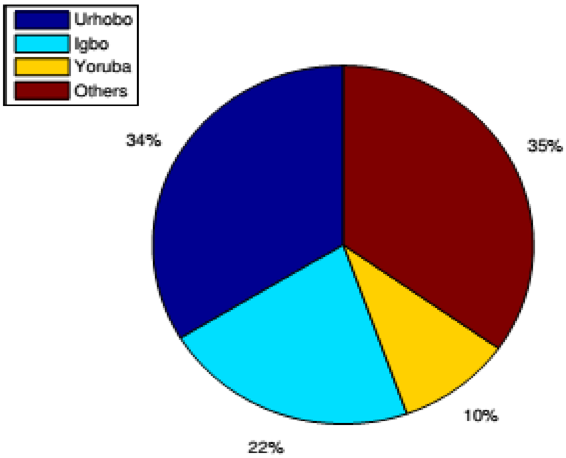

Figure 1: Pie chart showing the distribution of ethnic groups of respondents

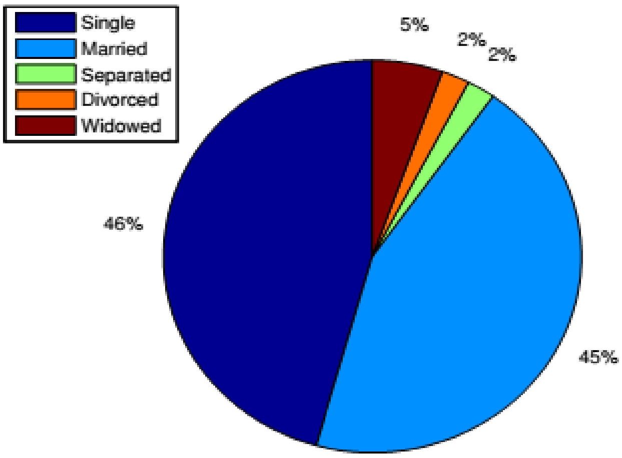

Figure 2: Pie chart showing the marital status of respondents

A close look at Table 2 revealed a general decline in the level of intergenerational transmission of the indigenous languages. While there was 100% transmission in the earlier generations ($\hat{p}_0 = 1$), the post-colonial era has seen the rise of English language, which is a Language of Wider Communication (LWC). A variant of English language, Pidgin English, has also contributed to the decline in intergenerational transmission. This could be seen from the estimates of the intergenerational transmission probability ($\hat{p}_1, \hat{p}_2$) for the various ethnic groups.

The value of $\hat{p}_1$ for the group of married respondents is shown interestingly to be close to 1 across all the ethnic groups under study. This shows that a greater proportion of the respondents who have children actually could communicate in their mother tongue, but due to one reason or the other, they do not bequeath their languages to their children. It is obvious that children from such families, when they become parents, will not be able to speak their heritage language to their kids, thus putting an end to that lineage of the family in terms of indigenous language ability.

Similarly, looking at $\hat{p}_1$ for the group of younger respondents, it is obvious that the intergenerational transmission probability is low, except for the Igbo tribe, with a value of 0.78. This is in line with the fact that many of the Igbo respondents in this category were born and raised in the community and their parents, more than any of the other ethnic groups, ensured that the children have sufficient ability to speak the Igbo language. The frequency of communication of the language at the home front is very high. In fact, in most of such families in the community, Igbo is the main language of communication in such homes. It may also not be unrelated to the fact that most of those respondents come from families in which both of the parents were of the same tribe, thereby reducing the possibility of conflict between parents on the language the growing child should imbibe (the father's or mother's).

The scenario is different with the Urhobo language in the community. This manifests in the low value of $\hat{p}_1$ (0.36) for the younger Urhobo respondents. Many Urhobo parents of the community prefer to communicate to their children in Pidgin English, clearly to the detriment of the children because they, in turn, cannot impart their language to their offspring when they start having children. Thus from that generation onwards, there will be zero transfers from parents to children, except the parents key in to revitalization programmes to enable them and their children to learn their indigenous language, or possibly get married to spouses of the same ethnic group who may impart the language to their offspring.

From the data presented in Table 1, the estimate of the mean age at marriage in the community was 33 years for males, 28 years for females and 30 years for both sexes. With the assumption that $\hat{p}_0 = 1$, it is possible to fit exponential curves on the intergenerational transmission probability ($\hat{p}_j$) against the time epochs, for the various languages surveyed, as captured in Figure 3.

The exponential curve is given as:

$$p_j = ae^{bt_j} \tag{21}$$

where a and b, the parameters of the exponential curve, may be estimated through the non-linear least squares procedure.

Analysis of Figure 3 revealed that the Igbo language in the community is sufficiently healthy, as shown by the curve, while the status of the Urhobo language is very low with the possibility of it becoming severely endangered as close as the fifth generation.

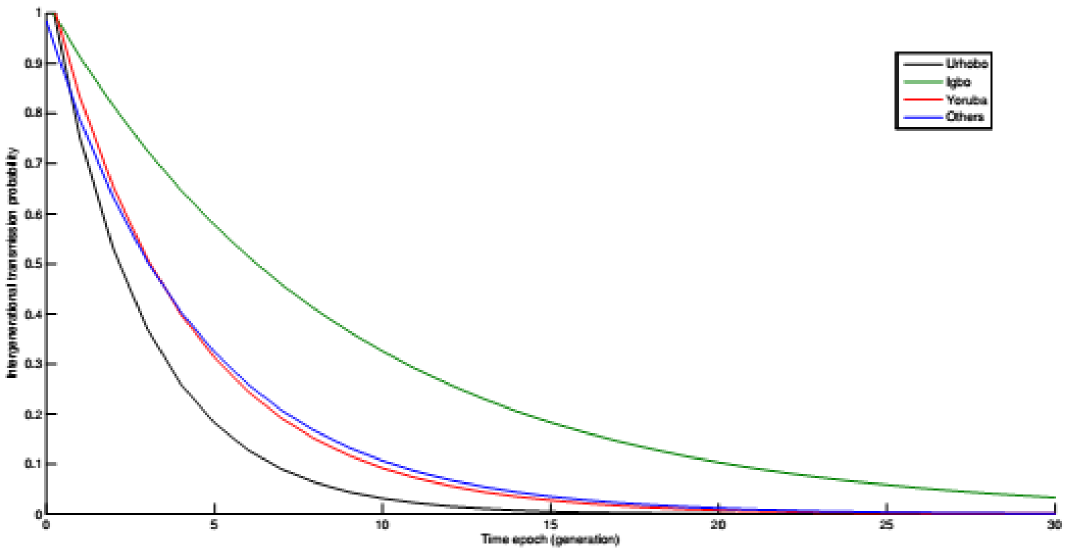

Figure 3: the estimated intergenerational transmission probabilities against time for the languages surveyed.

4.0 Discussion and Conclusion

From the study, it is quite clear that efforts should be made to ascertain the status of many indigenous languages, as the level of intergenerational transfer of language from parents to children should be sufficiently high for such languages to continue to be virile. Population-wise, Nigeria is growing, but in terms of indigenous language speakers, it is dwindling. The scenario in most families outside our study area is likely similar to that obtained in the community of our study. It should therefore be of importance to government to stem this tide of decline in order to maintain cultural identity as well as a multilingual society. Indigenous languages are iconic symbols of cultural identity and are also integral components of many multilingual societies. Furthermore, there is the need to include in national census questionnaires, brief questions to properly ascertain the ethnicity and level of intergenerational transmission of the heritage language among citizens of the country. This is a veritable means of eliciting relevant and up-to-date data that will enable planners to design language revitalization programmes.

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